

# Within-Job Wage Inequality: Performance Pay and Skill Match<sup>\*</sup>

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## Abstract

By decomposing residual wage inequality for the highly educated, I find that the within-job component is the main contributor to both the level and increase of wage inequality from 1990 to 2000. To explain this fact, I propose a model that allows within-job wage inequality to be influenced by performance-pay incidence and skill mismatch. Both factors were found to be positively correlated with within-job wage inequality. Performance pay amplifies ability dispersion through self-selection and work incentives; skill mismatch causes wage inequality even among individuals with the same ability level. I calibrate the model to the US economy in 1990 and quantify the importance of these two factors for wage inequality. The model explains around 71.5% of residual wage inequality for the high skill group in 2000. The skill-match channel explains 18.8% and performance-pay channel explains 34.1% of the increase in wage inequality.

**Keywords:** wage inequality, performance pay, skill match

**JEL Code:** I24, J24, J30, J31, J33

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# 1 Introduction

Wage inequality has been increasing rapidly since the 1970s. It has been documented that within group wage inequality or residual wage inequality counts for around 2/3 of total wage inequality; in other words, 2/3 of wage inequality cannot be explained by observed demographic characteristics.<sup>1</sup> In this paper, I will study this fact in greater detail and provide two channels for explain both the level and the change of wage inequality.

I compute residual wage inequality for both high (college and above) and low education groups from 1983 to 2013 and find the former had a higher level and increased faster, especially between 1990s and 2000s. Even if I control for more job characteristics including industry, occupation, firm size, location, citizenship and so on, the residual wage inequality decreases only by 10%. This suggests that the wage inequality within both the industry and occupation may contribute greatly to the total wage inequality. To further refine this, I define the job belonging to the category of industry and occupation and decompose both the level and change of wage inequality into the between-job and within-job components. The decomposition result shows that within-job inequality accounts for more than 80% of residual wage inequality between 1983 and 2013, and the contribution to the change ranges from 70% to 110% between 1990 and 2002. To my knowledge this fact has never been documented in the literature. Clearly it would be helpful to better understand residual wage inequality especially for the high education group.

In order to understand and explain this fact, I propose channels of performance pay and skill match within jobs. Workers in a performance-pay system are paid depending on how much they can produce; in the data, performance pay usually contains bonus, tips and commission. The counter part to this is the payment of a fixed hourly wage. The literature has documented that there is a positive wage effect of performance pay, I examine the relationship between within-job wage inequality and performance-pay incidence and find that there is a significant positive correlation; that is, jobs with a higher incidence of performance-pay position usually have a higher wage inequality. This suggests that the performance-pay channel is important to consider.

In addition, recent literature studies the skill match (mismatch) which measures the fitness between a worker and their job, as documented that skill mismatch may have negative effect on wages. I compute the skill mismatch in each occupation<sup>2</sup> by measuring the distance of skill requirement and acquirement and find that there is a positive correlation between skill mismatch and within-job wage inequality. This relationship is also significant when I measure the skill match by the relatedness between field of study and current job. This suggests skill match could

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<sup>1</sup>See, for example, Katz and Autor (1999).

<sup>2</sup>Since there is no industry data available, we compute skill mismatch in occupation under 2-digit code.

be another channel to explain within-job wage inequality. I build a model with these two channels and quantify their importance for residual wage inequality for highly educated workers.

In the model, each job contains two tasks under different payment systems: performance pay and fixed pay. Under fixed pay, people earn a pooling wage which is independent of individual effort and production. Under performance pay, workers get the output net monitoring cost. Jobs are heterogeneous in productivity and skill-match ratio; that is, in some jobs, workers have a high chance to be matched, but in some others the chance is very low. In addition, the match quality may be different; even though people find a job which matches his or her ability, their productivity from this match may be different. For instance, people with the same ability level who work in the same job could have different match quality due to factors like luck, personality and so on. In the model, the match quality is random, and distribution is the same across jobs.

Workers are heterogeneous in innate ability, and have choices over jobs, tasks and effort. In a fixed-pay task, workers sign a contract regarding earning and effort when they take the offer, while in the performance-pay task, workers decide how hard to work after observing the quality of the match. Regarding task choices, workers with low ability prefer a fixed-pay position while workers with high ability will choose to work under performance pay. In addition, in performance-pay tasks, workers with different ability level input different effort, in particular, high ability people will work harder than low ability ones. In sum, a worker's productivity depends on the following four factors: ability, effort, job productivity and skill-match quality.

Within-job wage inequality has the following two components: wage differences between performance-pay and fixed-pay tasks, and the wage dispersion within performance-pay tasks. Because of job sorting, the average wage in performance-pay task is higher than that in fixed-pay task, and this wage gap will depend on performance-pay incidence for each job. On the other hand, the wage dispersion in a performance-pay task is generated by productivity differences; therefore, it will be affected by effort and ability dispersion, and match quality distribution.

I first calibrate the model by matching the main facts on earnings and employment share in the US economy in 1990 and then compute the wage inequality in 2000 to quantify how much the model can explain. In a counterfactual analysis, I replace the value of parameters on performance pay and skill match in 2000 with those in 1990. In particular, I set the monitoring cost and match quality distribution parameter in 2000 to the same level as that in 1990. The main results will rely on two counterfactual analyses.

The equilibrium I focus on is the one with positive sorting. In particular, there is a cutoff ability in each job; and only if a worker's ability is higher than this value, will he or she choose a performance-pay task. In addition,

in a fixed-pay task, a worker feels indifferent between jobs, and in a performance-pay task, workers are sorted by ability; that is, people with a high (low) ability choose to work in a job with a high (low) ability cutoff. The quantitative results show that the model explains around 71.5% of residual wage inequality for high skill group in 2000, and the skill-match channel explains 18.8% and performance-pay channel explains 34.1% of the increase in wage inequality.

## **Related Literature**

**Wage inequality** A large number of studies document the trend of wage inequality that has generally been increasing since the 1970s. (e.g. Katz and Autor (1999), Card and DiNardo (2002), Autor et al. (2008), Acemoglu and Autor (2011), Piketty and Saez (2014), Beaudry et al. (2014) and Lee et al. (2015)). One classical theory on explaining increase of wage inequality is the change of skill premium due to skill biased technology change (SBTC). (e.g. Juhn et al. (1993), Krusell et al. (2000), Galor and Moav (2000), Shi (2002), Acemoglu (2003), Beaudry and Green (2005) and Burstein et al. (2015)). Literature on the high education group suggests that it is fruitful to study wage inequality within education group (e.g. Altonji et al. (2014)). Altonji et al. (2012) argue that earning difference across college majors can be larger than the skill premium between college and the high school.

Recent literature focuses on the decomposition of wage inequality. Barth et al. (2011) emphasize the role of plant difference within industry and argue that this could explain 2/3 of the wage inequality in the US. Card et al. (2013) show that plant heterogeneity and assortativeness between plants and worker explains a large part of the increase of wage inequality in West Germany. Mueller et al. (2015) study the skill premium within firms, and find that firm growth has contributed to the increase of wage inequality. Papageorgiou (2014) highlights the labor markets within firms and concludes that the within firm part might explain 12.5 percent to one third of the rise in wage inequality. Song et al. (2015), however, argue that the between-firm component is more important.

There is also a literature founded on the decomposition across occupations. While Kambourov and Manovskii (2009) argue that the variability of productivity shocks on occupations coupled with endogenous occupational mobility could account for most of the increase in within group wage inequality between the 1970s and middle 1990s, Scotese (2012) shows that changes in wage dispersion within occupation are quantitatively as important as wage change between occupations for explaining wage inequality between 1980 and 2000.

**Performance pay** Some literature on performance pay studies incentives and productivity. (e.g. Jensen and Murphy (1990), Lazear (2000)). Other literature explains the White-Black wage gap through the difference of tendency on performance pay across races (Heywood and Parent (2012)). The most relevant paper to our study is Lemieux et al. (2009). In their paper, the authors suggest performance pay as a channel through which the underlying changes in return to skill get translated into higher wage inequality. Their results show that 21% of the growth in the variance of wage can be explained between the late 1970s and the early 1990s. performance-pay position tends to be concentrated in the upper end of the wage distribution; for this reason, it provides a potential channel to study within group inequality.

**Skill match** The general idea of skill match is that people with the same characteristics might have different productivity from the job or machine they are working on. Violante (2002) provides a channel through vintage capital to decompose the residual wage inequality into worker's ability dispersion, machine's productivity dispersion and the correlation of these two. The author argues that this channel could explain most transitory wage inequality and 30% of the residual wage inequality. Jovanovic (2014) builds a model of learning by doing to emphasize the role of match between employees and employers; under this framework he discusses the role of improving signal quality and assignment efficiency.

In terms of measurement, there are generally two approaches in the literature. The first one is to measure the distance between skill requirement and acquirement based on scores of skills from NLSY79 and O\*NET (e.g. Sanders (2014), Guvenen et al. (2015), Lise and Postel-Vinay (2015)). The second approach is to measure job relatedness between field of study in the highest degree and current occupation from data in NSCG (e.g. Robst (2007), Arcidiacono (2004), Ritter and West (2014), Altonji et al. (2014) and Kirkebøen et al. (2014)).

**Organization of the paper** The paper is organized as follows: section 2 describes the data and presents main facts of wage inequality decomposition; section 3 builds a model with performance pay and skill match; section 4 describes the equilibrium and theoretical results; section 5 presents the quantitative result; section 6 has a discussion on multiple dimensions of ability and section 7 is the conclusion.

## 2 Facts

In this section, I document the facts on the wage inequality, performance pay and skill match. I compute wage inequality under different measurements and then decompose both the level and the change. For the performance pay, I present the facts that there is a positive correlation between performance-pay incidence and within-job wage inequality, and that jobs with high performance-pay incidence usually contribute greatly to the change of wage inequality. For the skill match, I document its relationship with both the wage and wage inequality under two alternative definitions.

### 2.1 Data

Data in this paper is from several sources: the March Current Population Survey (March CPS),<sup>3</sup> National Survey of College Graduates (NSCG), Panel Study of Income Dynamics (PSID), National Longitudinal Survey of Youth 1979 (NLSY79) and O\*NET. March CPS includes the longest high frequency data series enumerating labor force participation and earnings in the US economy. NSCG has information on relatedness between workers' fields of study and current occupations. PSID contains the information on earning in details including tips, commission, bonus people earned. NLSY79 provides information of workers' ability in different dimensions and O\*NET has occupational information on skill requirements.

Since Lemieux et al. (2009) have a comprehensive description on PSID, and Sanders (2014) and Guvenen et al. (2015) have a discussion on NLSY79 and O\*NET, I will mainly describe some main statistical features in the March CPS and NSCG.

**CPS** In March CPS, the education level is grouped under six categories: primary, high school dropout, high school graduate, some college, college graduate and post college. The highly educated includes workers who have college degree and above,<sup>4</sup> and the proportion of this group increased from 17% in 1983 to 34% in 2013. The schooling years are implied as 6, 9, 12, 14, 16, 18 for these groups respectively, and then the potential experience is computed as the year after graduation, that is,  $\max(\text{age} - \text{schooling} - 6, 0)$ . CEPR provides 2-digit and 3-digit occupation and industry code, but they are not time consistent through 1983 to 2013.<sup>5</sup> I build a 1-digit code on

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<sup>3</sup>I get the data from Center for Economic and Policy Research(CEPR).

<sup>4</sup>An alternative way is to include people with some college, the reason I don't use it is that the job match data in NSCG doesn't have information on that.

<sup>5</sup>For 2-digit code they are consistent in the following two sub-periods: 1983-2002 and 2003-2013.

industry and occupation as consistent with that in Lemieux et al. (2009). A consistent 3-digit code is built under the way proposed by Dorn (2009), and it is also used to group a consistent 2-digit code.

Only full time and full year workers<sup>6</sup> whose ages are between 16 and 65 are kept. The wage is the real hourly earnings, and I drop the earnings which are less than half of minimal wage in 1982 dollar or with top code defined by CEPR or higher than 1000.<sup>7</sup>

**NSCG** Every ten years, NSCG provides information on the relatedness between the field of study of the highest degree and the current occupation. It tracks people who reported having a college degree in census survey and asks how close the current occupation related with their fields of study. In the survey, the three possible responses to the question are close, some close and not at all. I take these three responses as the proxy of skill-match degree. In particular, the job relatedness is computed as the ratio of the amount of people who reported the closely relatedness in the survey to the total number of respondents. The calculation is weighted by the sample weight.

In addition, there are four levels of schooling year—16, 18, 19 and 21. They are regrouped into three levels: Bachelor (16), Maser (18,19) and PhD (21). The potential experience or tenure is calculated as the same way as that in the March CPS. The major code is regrouped under the category in the Department of Education. Occupation code is regrouped to be consistent with that in the March CPS. I keep only the full time workers with the age between 16 and 65 and drop the annual earnings which are higher than 4 million or less than 2, 800.

Table B.1 and B.2 present some statistic description of NSCG1993 and NSCG2003 which provide information in the year of 1990 and 2000 respectively. In the sample, the total observation are 94,360 in 1990 and 55,465 in 2000, and the average tenure are 19.11 and 20.92 respectively, the annual earnings are 67514.19 and 78042.82 under current year price, the overall inequality calculated by the variance of log annual earning has increased.

The proportion of job relatedness has not changed very much: the proportion of the closely related group is around 0.6. As shown in Table B.3, there is not much difference between gender and race groups. However, there is an increasing trend in education level. In particular, it increases from 0.5 for Bachelor to 0.88 for PhD in 1990 and the trend is similar in 2000.

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<sup>6</sup>Defined as those work at least 40 weeks in a year and 35 hours in a week.

<sup>7</sup>In some literature, this value is pretty low. For example, in Lemieux et al. (2009) it is 100, and in Acemoglu & Autor (2011), it is around 180. Since I will only focus on the highly educated, I want to keep as many observations as possible.

## 2.2 Wage inequality

In this subsection, I compute the residual wage inequality from the March CPS. I first measure the wage inequality as the variance of log hourly earnings. The left panel of Figure 2.1 documents this fact by education group from 1983 to 2013. Generally speaking, the wage inequality has been increasing since the 1980s; however, the patterns for different education groups are different. Comparing to low education group, the highly educated has a higher level and also increases faster especially in the late 1990s.

To control for the observed characteristics, I compute the residual wage inequality based on the regression

$$\ln wage = constant + i.education * i.exp + i.gender + i.race + \epsilon.$$

Specifically, I regress log hourly earnings ( $\ln wage$ ) on demographic characteristics including education ( $edu$ ), potential experience ( $exp$ ), gender and race. Then the residual wage inequality is computed as the variance of the residues, that is,  $Var(\epsilon)$ . As shown on the right panel of Figure 2.1, the residual wage inequality has a similar pattern as the wage inequality from raw data. As a robustness check, I calculate the wage inequality as the Gini coefficient and 90/10 ratio. Figure B.1 shows that the Gini coefficient has similar pattern as the residue variance. For 90/10 ratio, although the pattern is different, high education group has similar pattern.

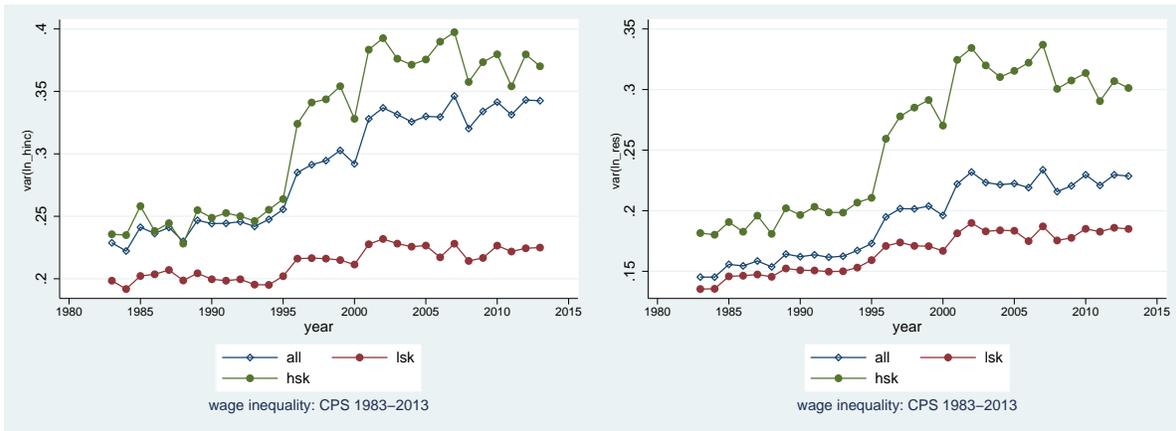


Figure 2.1: wage inequality by education group

The above results suggest the specialty of the highly educated, hence in the paper I only focus on this group. The left panel of Figure 2.2 compares the total wage inequality and the residual wage inequality. It shows that the residual wage inequality accounts around 80% of total wage inequality in this group. The number is higher than 2/3 as documented in the literature for the whole sample. What is more, the right panel documents the trend

of residual wage inequality when controlling more job characteristics including industry, occupation, location, firm size, citizenship and so on. It shows that controlling industry and occupation could explain 10% more, but the result doesn't change much when more variables are controlled. These facts suggest that the wage inequality within industry and occupation may have high contribution to the total wage inequality. To refine this, I decompose the residual wage inequality by an accounting exercise in the next subsection.

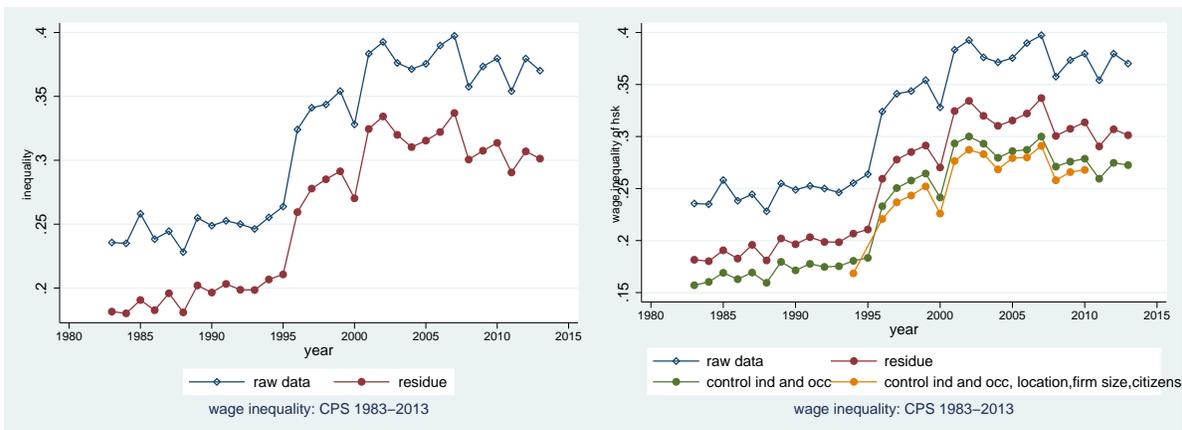


Figure 2.2: residual wage inequality for high education group

### 2.3 Decomposition

In this subsection, I decompose both the level and change of the residual wage inequality into two components: within job and between job. A job is defined as a pair of industry and occupation. Specifically, in each industry there are different occupations and for the same occupation it will be in different industries, then I define occupations in different industries as different jobs. Under this definition, for example, if there are 10 industries and in each industry there are 8 occupations, then there are 80 jobs in total. One of benefits of this definition is to solve the mis-classification problem. In particular, the content of either occupation or industry may have changed over 30 year especially under 2 or 3-digit code, and some of them might even disappear. But for 1-digit code it is more likely to be consistent, and when incorporating both the industry and occupation, I can get a consistent job code as well as a large sample of jobs.

**Decomposition of the level** Suppose there are  $N$  jobs, for job  $n = 1, \dots, N$ , let  $P_n$  the employment share,  $V_n$  the wage inequality, and  $E_n$  the average earnings. Then  $\sum_n P_n V_n$  is the weighted average of within-job wage inequality, and  $\sum_n P_n (\ln E_n - \sum_{n'} P_{n'} \ln E_{n'})^2$  is the weighted average of between-job wage inequality where  $\sum_{n'} P_{n'} \ln E_{n'}$  is

the average log earnings in the economy. Then the total wage inequality  $var(\ln E)$  can be decomposed into the between-job and within-job components as follows

$$var(\ln E) = \sum_n P_n V_n + \sum_n P_n (\ln E_n - \sum_{n'} P_{n'} \ln E_{n'})^2.$$

I then compute the contribution of within-job wage inequality as the ratio of the weighted average of inequality across jobs  $\sum_n P_n V_n$  to the total wage inequality  $var(\ln E)$ , and the contribution of between-job wage inequality is the ratio of the weighted average of between-job wage inequality to the total wage inequality. It is shown in Figure 2.3 that the contribution of within-job inequality is persistently large. In particular, it is around 85% under the code of 1-digit industry and 1-digit occupation as shown on the left panel, and this value is 80% under the code of the 1-digit industry and 2-digit occupation shown on the right panel.

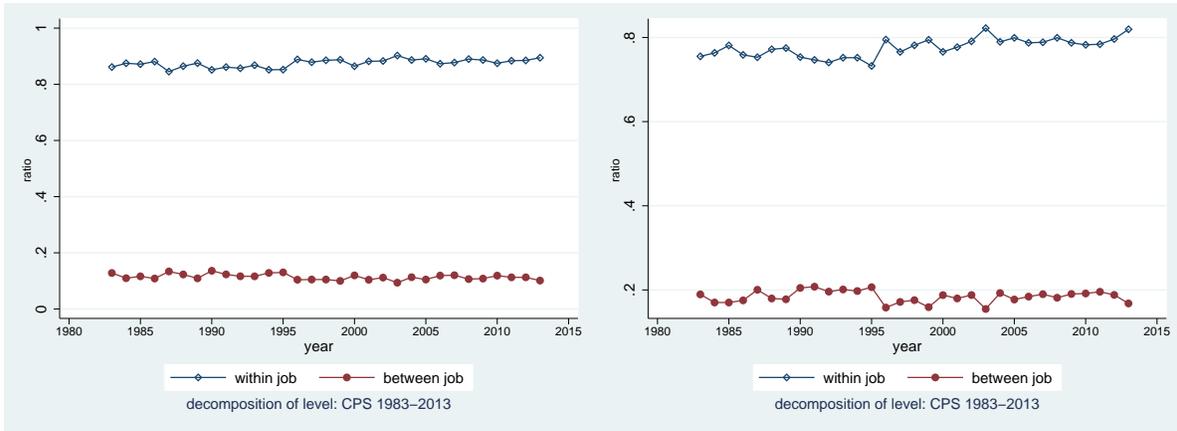


Figure 2.3: Decomposition of wage inequality

**Decomposition of the change** In addition to the level, I also decompose the change of wage inequality into job related components. In job  $n$  at year  $t$ , let  $V_{n,t}$  the wage inequality,  $E_{n,t}$  the average log earnings,  $P_{n,t}$  the employment share and  $E_t$  the average earning among all the jobs. Then the change of within-job wage inequality is  $V_{n,t+1} - V_{n,t}$ , the change of between-job wage inequality is  $(E_{t+1} - E_{n,t+1})^2 - (E_t - E_{n,t})^2$ , and the change of employment share is  $P_{n,t+1} - P_{n,t}$ . Therefore, the change of wage inequality between year  $t + 1$  and year  $t$ ,  $V_{t+1} - V_t$ , can be decomposed into four components: the weighted average change of within-job wage inequality  $\sum_{n=1}^N P_{n,t} [V_{n,t+1} - V_{n,t}]$ , the weighted average change of between-job wage inequality  $\sum_{n=1}^N P_{n,t} [(E_{t+1} - E_{n,t+1})^2 - (E_t - E_{n,t})^2]$ , the weighted average change of employment share  $\sum_{n=1}^N (P_{n,t+1} - P_{n,t}) [V_{n,t} + (E_t - E_{n,t})^2]$  and interaction terms which are the

products of changes of employment share and the changes of the sum of within and between-job wage inequalities.

$$\sum_{n=1}^N (P_{n,t+1} - P_{n,t}) \{ (V_{n,t+1} - V_{n,t}) + [(E_{t+1} - E_{n,t+1})^2 - (E_t - E_{n,t})^2] \},$$

Formally, I decompose the change of wage inequality as follows

$$\begin{aligned} V_{t+1} - V_t &= \sum_{n=1}^N P_{n,t} [V_{n,t+1} - V_{n,t}] \\ &+ \sum_{n=1}^N P_{n,t} [(E_{t+1} - E_{n,t+1})^2 - (E_t - E_{n,t})^2] \\ &+ \sum_{n=1}^N (P_{n,t+1} - P_{n,t}) [V_{n,t} + (E_t - E_{n,t})^2] \\ &+ \sum_{n=1}^N (P_{n,t+1} - P_{n,t}) \{ (V_{n,t+1} - V_{n,t}) + [(E_{t+1} - E_{n,t+1})^2 - (E_t - E_{n,t})^2] \}. \end{aligned}$$

Similarly, the contribution of each component is defined as the ratio of its change to the total change in wage inequality. The left panel of Figure 2.4 presents the result between 1990 and 2002 under 1-digit code where the base year is 1990. It shows that the within-job component has persistently high contribution to the change of wage inequality. As a robustness check, the right panel reports the decomposition result under 1-digit industry and 2-digit occupation, and the contribution of within-job inequality is still the highest.

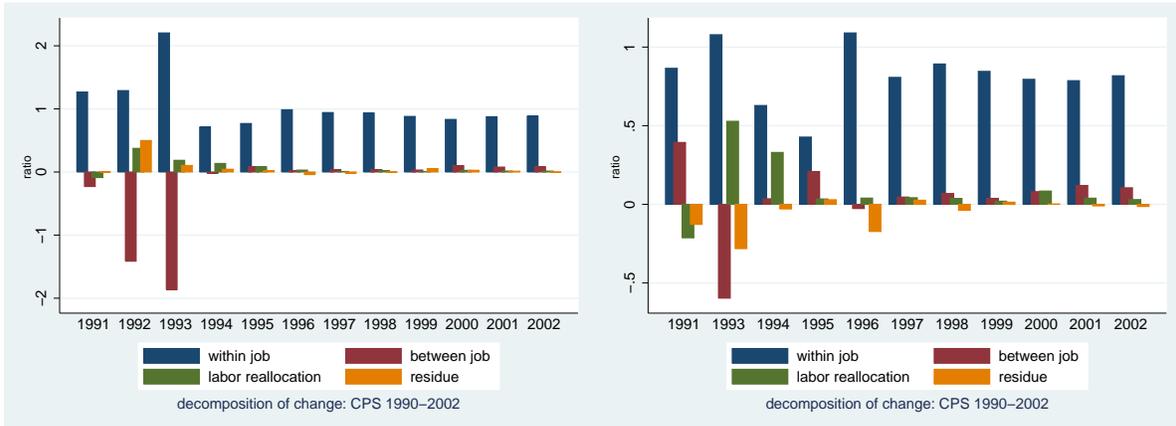


Figure 2.4: Decomposition of the change of wage inequality

## 2.4 Performance pay

It has been shown that the wage in performance-pay position is generally higher than that in the fixed-pay position.<sup>8</sup> In this subsection, I document the facts on performance pay and wage inequality. In the literature, performance-pay incidence describes how likely the job will provide performance-pay position. Lemieux et al. (2009) calculate the performance-pay incidence for different jobs based on a regression of which the result is copied in Table B.5. I follow their way and keep the year up to 1990 and 2000 to predict the performance-pay incidence in these two years respectively. Figure 2.5 plots the results together with within-job wage inequality in 1990 and 2000. In the picture, each point represents a job, and the horizontal line shows that performance-pay incidence and the vertical line is the wage inequality in this job. It shows that there is a significant positive relationship between the performance-pay incidence and within-job wage inequality; in other words, jobs with high performance-pay incidence usually have high within-job wage inequality.

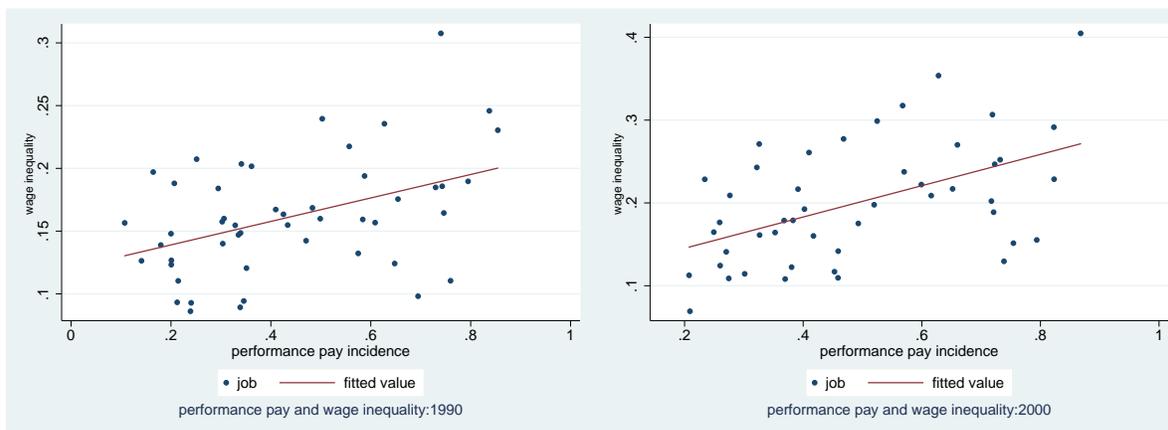


Figure 2.5: Performance pay and wage inequality

In addition to the level, in Table 2.1, I tabulate the jobs which generally have high contribution to the change of wage inequality. In particular, I find the jobs with the contribution in top 5 every year and then count the frequency found in that time period. For example, the job defined as the pair of industry “Fin.insur.,&real est.” and occupation “Sales” has the contribution in the top 5 for 6 times out of 12. Relating this fact to performance-pay incidence, I find that jobs which have high contribution to the change of wage inequality usually have high incidence of performance pay.

<sup>8</sup>See, for example, Lemieux et al. (2009).

Table 2.1: Contribution in top 5:1990-2002

ind desc.	occ desc.	#/12
Bus.&prof. service	Managers	12
Bus.&prof. service	Professionals	11
Retail trade	Sales	9
Fin.,insurance.,&real est. service	Sales	6
Fin.,insurance.,&real est. service	Managers	5

## 2.5 Skill match

In the literature, skill mismatch (match) usually has two measurements: one is the distance between skill requirement from jobs and acquirement from workers; the other one is the job relatedness between the field of study of the highest degree and the current occupation. In this subsection, I document the relationship between skill mismatch and both the wage and wage inequality.

**Wage effect** Under the first definition, Guvenen et al. (2015) show that skill mismatch has significant and persistent negative effect on the wages and earnings as copied in Table B.6.

Under the second definition, I estimate the wage effect based on a regression. In particular, I regress the log annual earnings on skill match quality, demographic characteristics, occupational characteristics, major and other factors, that is

$$l\text{earnings}_{ijm} = \beta D_i + \alpha Z_j + \theta M_m + \delta_1 \text{close}_{jm} + \delta_2 \text{some}_{jm} + \gamma X_i + \epsilon_{ijm},$$

where  $l\text{earnings}_{ijm}$  is the log earnings of worker  $i$  in occupation  $j$  and major  $m$ ,  $D$  includes a vector of demographic variables (tenure, age, gender, race and etc.),  $Z$  denotes the occupation,  $M$  denotes the major,  $\text{close}$  and  $\text{some}$  denote the job is closely and some related respectively,  $X$  includes all other factors: parents education, degree location, work location and so on.

Table B.4 presents part of the regression results, where  $\delta_1 = 0.171, \delta_2 = 0.118$  in 1990 and  $\delta_1 = 0.229, \delta_2 = 0.170$  in 2000. The result that  $\delta_1 > \delta_2 > 0$  suggests that mismatch has significant negative effect on earnings. In other words, comparing to the case of mismatch (not related at all), match group have 17.1 percent higher in annual earning in 1990 and in 2000 this value is 22.9 percent. What is more, the effect of job match is getting larger in 2000 than that in 1990.

**Wage inequality effect** To show the relationship between skill match and wage inequality, I first measure skill mismatch for the highly educated following the way in Guvenen et al. (2015). Instead of regressing on wage to estimate the weights on different types of skills, I use the equal weights. Then I compute the distance between skill requirements from O\*NET and skill acquirement from NLSY79. Since O\*NET has only occupational information on skills, the skill mismatch is computed under 2-digit occupational code. Figure 2.6 plots the skill mismatch and wage inequality across occupations, where horizontal line shows the level of mismatch and the vertical line is the wage inequality. It shows that there is a positive relationship in both 1990 and 2000. More importantly, the relationship is getting more significant in 2000.

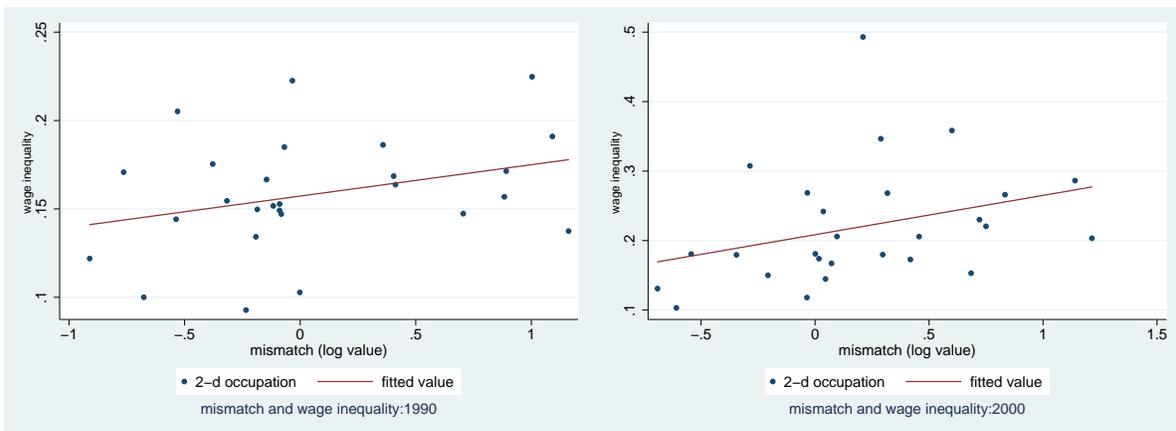


Figure 2.6: Skill mismatch and wage inequality

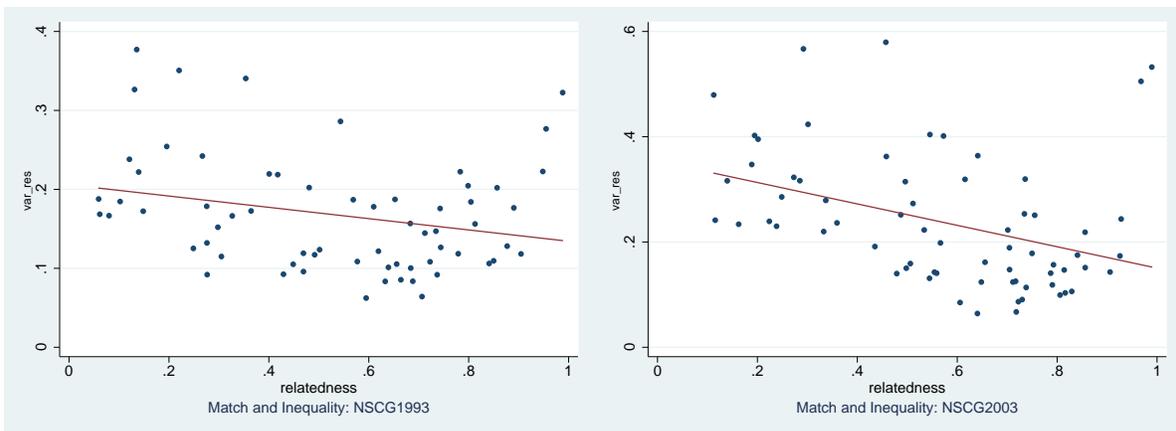


Figure 2.7: Job relatedness and wage inequality

In addition, Figure 2.7 plots job relatedness and wage inequality across occupations under 3-digit code. In this figure, each point represents one occupation and the wage inequality is the residual wage inequality in NSCG. It

shows that there is a significant negative relationship between job relatedness and the within-job wage inequality. Since high job relatedness implies low mismatch, this result is consistent with the previous one that there is a positive relationship between mismatch and wage inequality.

### 3 The model

**Environment** In the model, a job is characterized by a pair of industry and occupation, and there are  $I$  industries and  $J$  occupations, hence  $I \cdot J$  jobs in total. In each job, there are two kinds of tasks: fixed-pay task  $FP$  and performance-pay task  $PP$ . Jobs are different in both the productivity  $A_{ij}$  and skill match probability  $p_{ij}$ . Workers are heterogeneous in innate ability  $a$  and will choose jobs, tasks and efforts to maximize utility. Job characteristics  $A_{ij}$ ,  $p_{ij}$  and worker's ability and its distribution  $G(a)$  are public information, but for skill match quality only the distribution  $F(\cdot)$  is known.

**Human capital** A worker's efficient labor depends on her or his ability  $a$ , job-specific productivity  $A$ , skill match quality  $\eta$ , and the effort  $e$ . For simplicity, the efficient labor is assumed to be linear in these factors, that is,  $h = Aa\eta e$ . Ability  $a$  follows Pareto distribution  $a \sim G(a) = Pd(\theta_a)$ ,  $a \geq 1, \theta_a > 2$ , where the lowest level of ability is 1 and  $\theta_a > 2$  is to guarantee the existence of variance. There is a probability  $p$  that the worker's ability will be matched with the job. Given matched there is a skill match quality  $s$  which also follows Pareto distribution, and if there is no match then the skill match quality is 1. Formally, the skill match quality  $\eta$  is

$$\eta = \begin{cases} s & \text{with probability } p \\ 1 & \text{with probability } 1 - p \end{cases}, \text{ and } s \sim F(s) = Pd(\theta_s), s \geq 1, \theta_s > 2,$$

where  $\theta_s$  captures the dispersion degree of match quality, and the assumption of  $\theta_s > 2$  is made in order to have bounded variance. In this baseline model, the skill match probability  $p$  is exogenous and job specific.

**Jobs** In the model, there is one representative final goods producer which includes all the industries and occupations, and the total output is the product of output across industries. In particular, let  $Y$  is the total output and  $Y_i$  is the output in industry  $i$ , then  $Y = \prod_{i=1}^I Y_i^{\beta_i}$  where  $\beta_i$  is the share of industry  $i$  with  $\sum \beta_i = 1$ . In addition, output in each industry  $i$  is the CES aggregate across all occupations in this industry  $Y_{ij}$ , that is,  $Y_i = (\sum_{j=1}^J Y_{ij}^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}$ , where

$\sigma$  is the elasticity of substitution across occupations within industry. The production function in job  $(i, j)$  is the CES aggregate of efficient labor of two tasks. Let  $H_{ijF}$  and  $H_{ijP}$  are the total efficient labor in task  $FP$  and  $PP$  respectively, then the production function in this job is

$$Y_{ij} = (H_{ijF}^\gamma + H_{ijP}^\gamma)^{\frac{\mu}{\gamma}},$$

where  $\frac{1}{1-\gamma}$  is the elasticity across tasks and  $\mu$  is the labor share. Let  $D_{ijF}$  and  $D_{ijP}$  the ability domain of workers in task  $FP$  and  $PP$  respectively. Denote  $H(a, \eta)$  the joint distribution of ability and match quality,  $h_{ijF}$  and  $h_{ijP}$  the human capital of an individual from task  $F$  and  $P$  in job  $(i, j)$ , then the total efficient labor in task  $FP$  and  $PP$  are

$$H_{ijF} = \int_{a \in D_{ijF}} \int_{\eta} h_{ijF}(a, \eta) dH(a, \eta),$$

and

$$H_{ijP} = \int_{a \in D_{ijP}} \int_{\eta} h_{ijP}(a, \eta) dH(a, \eta)$$

respectively. Let  $H(\eta)$  the distribution of the skill match quality, then  $H(\eta) = (1-p) + pF(\eta)$ . In the benchmark, the ability and skill match quality are independent, that is,  $H(a, \eta) = G(a)H(\eta)$ .

In each job  $(i, j)$  there is a wage rate  $w_{ij}$  for one unit of efficient labor. In addition, there is an extra cost  $c_{ij}$  on creating a vacancy in performance-pay position other than wage rate  $w_{ij}$ . Given this wage rate, the representative final goods producer chooses labor allocation across jobs and tasks to maximize profit

$$\max_{\{D_{ijF}, D_{ijP}\}} Y - \sum_{i,j} [H_{ijF} + c_{ij}H_{ijP}]w_{ij}.$$

**Workers** A worker cares about consumption  $c$  and working effort  $e$ . In particular, the utility function is linear in consumption and has a quadratic form on effort, that is,  $U(c, e) = c - \frac{1}{2}be^2$ .  $b$  measures the degree of disutility on effort, and consumption  $c$  will equal to total earnings. The earning in task  $FP$  is  $\bar{E}$  in the contract, but in task  $PP$ , worker will get all the product net monitoring cost  $M$  defined in efficient labor unit. Denote  $w$  the wage rate for one unit of efficient labor, then the hourly earning is the product of net efficient labor and unit wage rate, that is,  $E = (h - M)w$  or  $E = (Aa\eta e - M)w$ .

Workers will make choices on jobs, tasks and efforts. As shown in equation (3.1), a worker observes job's

productivity  $A_{ij}$  and skill match probability  $p_{ij}$  and then chooses job and task. In task  $PP$ , he or she chooses the effort  $e_{ij}(a)$  after the skill match quality is realized, but in task  $FP$ , both the earnings and effort  $(\bar{E}_{ij}, \bar{e}_{ij})$  have to be decided at the beginning.

$$\{A_{ij}, p_{ij}\} \longrightarrow [(i, j) \& (P/F)] \begin{cases} P & \xrightarrow{\{\eta_{ij}\}} [e_{ij}(a)] \longrightarrow E_{ij}(a) \\ F & \rightarrow (\bar{E}_{ij}, \bar{e}_{ij}) \xrightarrow{\{\eta_{ij}\}} \bar{E}_{ij} \end{cases} \quad (3.1)$$

More specifically, in job  $(i, j)$  and task  $FP$ , the pooling earning  $\bar{E}$  is based on the expectation on worker's human capital, that is,

$$\bar{E} = \int_{a \in D_{ijF}} A_{ij} a \eta \bar{e} w_{ij} dH(a, \eta)$$

hence it is a function of effort, and workers just need to choose the effort. In particular

$$\begin{aligned} EU_{ij}^F &= \max_e c - \frac{1}{2} b \bar{e}^2 \\ st. c &= \bar{E} \\ \bar{E} &= E(A_{ij} \eta_{ij} a | a \in D_{ijF}, \eta) \bar{e} w_{ij}. \end{aligned}$$

In the task  $PP$ , workers choose the effort after the match quality is realized. In particular

$$\begin{aligned} U_{ij}^P(a; \eta_{ij}) &= \max_e c - \frac{1}{2} b e^2 \\ st. c &= (A_{ij} a \eta_{ij} e - M) w_{ij} \end{aligned}$$

Then, at the beginning, a worker chooses the job and task which gives the highest expected utility.

$$EV(a) = \max_{\{(i, j) \& (P/F)\}} \{EU_{ij}^F, EU_{ij}^P(a)\}.$$

## 4 Equilibrium

### 4.1 Definition

The equilibrium in labor market is described by the wage rate  $\{w_{ij}\}$  and the labor allocation across jobs and tasks  $\{D_{ijF}, D_{ijP}\}$  and it is defined in a standard way.

Given the wage rate, the representative final goods producer chooses labor allocation across jobs and tasks to maximize the profit

$$\max_{\{D_{ijF}, D_{ijP}\}} Y - \sum_{i,j} [H_{ijF} + c_{ij}H_{ijP}]w_{ij},$$

where

$$\begin{aligned} H_{ijF} &= \int_{a \in D_{ijF}} \int_{\eta} h_{ijF}(a, \eta) dH(a, \eta) \\ h_{ijF}(a, \eta) &= A_{ij} a \eta \bar{e}_{ij} \end{aligned}$$

and

$$\begin{aligned} H_{ijP} &= \int_{a \in D_{ijP}} \int_{\eta} h_{ijP}(a, \eta) dH(a, \eta) \\ h_{ijP}(a, \eta) &= A_{ij} a \eta e_{ij}(a, \eta), \end{aligned}$$

and  $\bar{e}_{ij}$  and  $e_{ij}(a, \eta)$  are the effort levels in *FP* and *PP* respectively.

On the other hand, given these wage rates, workers choose jobs, tasks and efforts to maximize the expected utility

$$EV(a) = \max_{\{(i,j), F/P\}} \{EU_{ij}^F, EU_{ij}^P(a)\},$$

where

$$\begin{aligned} EU_{ij}^F &= \max_{\bar{e}} U(\bar{E}, \bar{e}) \\ s.t. \bar{E} &= \int_{a \in D_{ijF}} A_{ij} a \eta \bar{e} w_{ij} dH(a, \eta) \end{aligned}$$

and

$$EU_{ij}^P = EU_{ij}^P(a | \eta), U_{ij}^P(a | \eta) = \max_e U(E, e).$$

Finally, the labor market clear condition requires  $\sum_{i,j} \int_{a \in D_{ij}} dG(a) = 1$ , where the total amount of labor force is assumed to be 1. The solution details are provided in Appendix A.

## 4.2 An equilibrium with positive sorting

The model may have multiple equilibria. In particular, jobs with high productivity may have some workers with low ability or low productivity jobs may have workers with high ability. In the benchmark, I only focus on the equilibrium with positive sorting. Specifically, there is a cutoff ability in each job and only if worker's ability is higher than this value will she or he choose performance-pay task. In addition, in the fixed-pay task, workers feel indifferent between different jobs. In the performance-pay task, workers are sorted by ability; that is, people with high (low) ability choose to work in a job with high (low) ability cutoff.

### Cutoff ability

In task *PP*, the expected utility is

$$EU^P(a) = \frac{(Aaw)^2}{2b} E(\eta^2) - Mw,$$

which is increasing in ability  $a$ . In task *FP*, workers have the same wage. Denote the domain of abilities of workers who work in fixed-pay position as  $D_F$ , then the expected utility is

$$EU^F = \frac{[E(A\eta a | a \in D_F, \eta)w]^2}{2b}$$

which is independent of ability  $a$ . Hence there is a cutoff ability  $a_{ij}^*$  such that worker in job  $(i, j)$  will choose task *P* only if  $a \geq a_{ij}^*$ . In addition, the cutoffs can be ranked and relabeled as  $\{a_n^*\}$ , such that  $a_1^* \leq \dots \leq a_n^* \leq a_{n+1}^* \leq \dots \leq a_N^*$ . Then there are some properties regarding those cutoff abilities and job choices.

**Equalization of expected utility in task *FP*** Workers with ability  $a$  such that  $a < a_1^*$  will work in the task *FP*. Since there is free labor mobility, the expected utility of task *FP* should be equalized among jobs, that is,  $E_n U^F = EU^F$ , for any job  $n = 1, \dots, N$ .

**Monotonicity of expected utility** Workers with ability  $a$  such that  $a_n^* \leq a < a_{n+1}^*$  can choose to work in job  $n + 1$  or the job with higher cutoff abilities but only be in the fixed-pay position; or they can work in job  $n$  or the job with

lower cutoff abilities in the performance-pay position. Since  $EU^P(a)$  increases in  $a$  and higher ability workers can always get at least as same as the lower ability workers, there is a monotonicity of the expected utility

$$E_{n+1}U^P(a) \geq E_nU^P(a), \text{ for } a \geq a_{n+1}^*, n = 1, \dots, N-1.$$

$$E_1U^P(a) \geq EU^F, \text{ for } a \geq a_1^*$$

**Continuity of expected utility** Because of continuity of utility function, the marginal worker with ability  $a_{n+1}^*$  is indifferent between job  $n$  and  $n+1$  in task  $PP$ , that is,

$$E_{n+1}U^P(a_{n+1}^*) = E_nU^P(a_{n+1}^*) \text{ for } 1 \leq n \leq N-1.$$

In addition, the marginal worker with ability  $a_1^*$  is indifferent between task  $FP$  in any job and task  $PP$  in the job with the lowest cutoff ability, that is,  $E_1U^P(a_1^*) = EU^F$ .

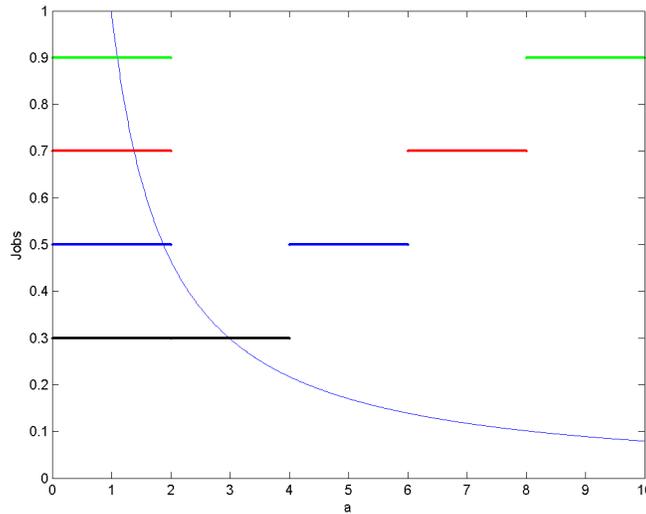


Figure 4.1: Job ladders

**An illustration of job choice** To illustrate above properties, I make a simple example in Figure 4.1. There are four jobs (four different colors of horizontal line in the figure), and hence there are four cutoff abilities which, in the picture, are  $a_{min} = a_1^* = 2$ ,  $a_2^* = 4$ ,  $a_3^* = 6$ ,  $a_4^* = 8$ . Then workers will be indifferent between jobs in the fixed-pay task if  $a < a_1^*$ , and choose job  $n$  with performance pay if  $a_n^* \leq a < a_{n+1}^*$ ,  $n = 1, 2, 3, 4$ . Therefore, in each job there

are some workers in the performance-pay task and some others in the fixed-pay task.

## Wage inequality

Wage inequality in the model is computed as the variance of log hourly earnings. Let  $E_{ij}(a)$  the hourly earnings for worker with ability  $a$  in job  $(i, j)$ ,  $\ln E_{ij}(a)$  the log value, and  $\ln \bar{E}$  the average value of log earnings. In addition, let  $\ln \bar{E}_n^F$  the log value of earning in job  $n$  under fixed pay and  $\ln E_n^P(a)$  the log value of earnings in job  $n$  under performance pay. And let  $N_n$  the employment share in job  $n$ ,  $N_{Pn}$  the proportion of worker under performance pay in job  $n$ . Then the average value of log earning can be computed as

$$\ln \bar{E} = \sum_{n=1}^N [(1 - N_{Pn})N_n \ln \bar{E}_n^F + \int_{a_n^*}^{a_{n+1}^*} \int_{\eta} \ln E_n^P(a) dH(a, \eta)].$$

Then the wage inequality can be computed as the following way

$$Var = \sum_{n=1}^N \{(1 - N_{Pn})N_n (\ln \bar{E}_n^F - \ln \bar{E})^2 + \int_{a_n^*}^{a_{n+1}^*} \int_{\eta} [\ln E_n^P(a) - \ln \bar{E}]^2 dH(a, \eta)\}$$

## 5 Quantitative analysis

### 5.1 Calibration

Parameters in this model include  $\{A_{ij}\}, \{c_{ij}\}, \{\beta_i\}, \gamma, \mu, \sigma, b, M, \theta_a, \theta_s$ . I compute the following first moments in each job  $(i, j)$ : the employment ratio ( $n_{ij}$ ), the performance-pay incidence ( $nP_{ij}$ ), the skill match ratio ( $p_{ij}$ ), and the average earnings ( $\{E_{ij}\}$ ). In addition, I compute the following first moments in each industry or occupation: the employment ratio ( $n_i$  and  $n_j$ ) and the average earnings ( $E_i$  and  $E_j$ ). Then I target these moments as well as the wage inequality in 1990 to calibrate the parameters.

I set the labor share  $\mu = 0.6$ , and calibrate  $\{\beta_i\}$  by targeting average labor earning across industries, that is,  $\beta_i = \frac{E_i}{\sum_i E_i}$ . Then I calibrate  $\theta_a, \theta_s, b, M, \sigma, \gamma$  in the following way.

Firstly, I rank cutoff ability  $(i_n, j_n) \sim n$  and normalize  $A_1 = 1$  in 1990. Theoretically, the rank of cutoffs is an equilibrium result, but to simplify the calculation, I sort the cutoffs by the average earnings. Hence a job with high average earnings in the data has a high rank of cutoff.

Secondly, I solve the cutoffs  $\{a_n^*\}$ , wage rate  $\{w_n\}$ , relative cost of job creation in performance-pay position

$\{c_n\}$ , and job specific productivity  $\{A_n\}$ . By targeting the match ratio  $\{p_n\}$  and performance-pay incidence  $\{nP_n\}$  in each job, equation (A.8) - (A.11) will pin down these variables. Match ratio data is from NSCG1993, since only occupational information is available, I assume match ratios are the same across industries within occupation. Performance-pay incidence data is from PSID based on the results from Lemieux et al. (2009).

Lastly, I target  $\{n_i\}, \{n_j\}, \{n_{ij}\}, \{E_j\}, \{E_{ij}\}$  and  $var(lnw)$  in 1990 and choose  $\theta_a, \theta_s, b, M, \sigma, \gamma$  to minimize sum of error squares.

Table 5.1: Parameters in benchmark model

Parameters	Descriptions	Value	comments
$\mu$	labor share	0.6	
$\sigma$	ES of occupations within industry	4.1	< 5 in Hsieh&Klenow (2009)
$\gamma$	ES between <i>PP</i> and <i>FP</i>	0.4	substitute if $\gamma < 1$
$\theta_a$	scale parameter in ability distribution	10.1	only college graduates
$\theta_s$	scale parameter in match quality	9.5	
$b$	disutility of effort	1.9	
$M$	monitoring cost	0.0035	1.17% of hourly earning
$Var(lnw)$	data	model	
1990	0.201	0.187 (93%)	
2000	0.284	0.203 (71.5%)	Parameters in 1990

**Result** Table 5.1 presents the calibration results in the benchmark model. Note that  $\theta_a$  is relative large compared to the value in the literature, however, given that the data only includes the highly educated and the wage inequality data is about the residue, this high value is reasonable. In the table,  $\theta_s$  is also high. However, since skill match quality is captured by both the  $\theta_s$  and skill match ratio  $\{p_n\}$ , skill match could have large variety even with this high value in  $\theta_s$ .  $M$  captures the monitoring cost in the unit of efficient labor and  $M = 0.0035$  implies the monitoring cost equals to 1.17% of the hourly earning in 1990. In the calibration,  $\sigma = 4.1$  is consistent with that in Hsieh and Klenow (2009) who argue this value should be smaller than 5.  $\gamma = 0.4$  implies the elasticity of substitution equals to 1.67, and it suggests that performance-pay task and fixed-pay task are substitute.

The last two rows of Table 5.1 show that the wage inequality is 0.201 in the year of 1990 and the model generate 0.187. This result shows that our model captures 93% of wage inequality in the data. I use the parameters  $\theta_a, \theta_s, b, M, \sigma, \gamma$  and  $\{\beta_i\}$  calibrated to compute  $\{A_{ij}\}, \{c_{ij}\}$  by targeting earning and employment ratio across jobs

in 2000, and then to compute the wage inequality. It shows that the model can generate inequality of 0.203 while in the data it is 0.284, hence the model captures 71.5% of wage inequality in 2000.

## 5.2 Comparative statics

In this subsection, I do comparative statics on  $b, M, \theta_s, \theta_a$ . As shown in Figure 5.1, there is a negative relationship between wage inequality and  $b$  or  $M$ . Since low  $b$  implies low disutility of effort, it will motivate workers in task PP to work hard. Hence it will generate high wage difference between performance-pay task and fixed-pay task, and also high level wage inequality within performance-pay task. When  $M$  is small, many people choose to work in task PP due to low monitoring cost, hence it may increase wage inequality within the group of task in PP.

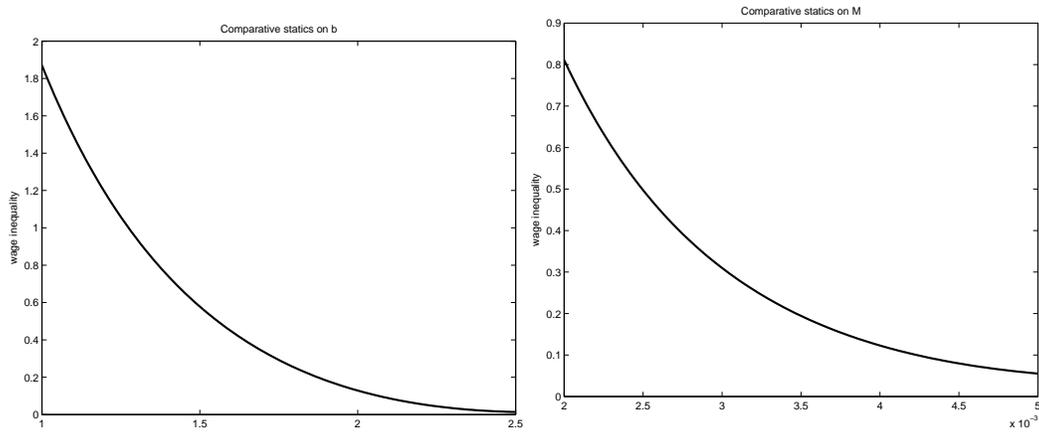


Figure 5.1: Comparative statics on  $b$  and  $M$

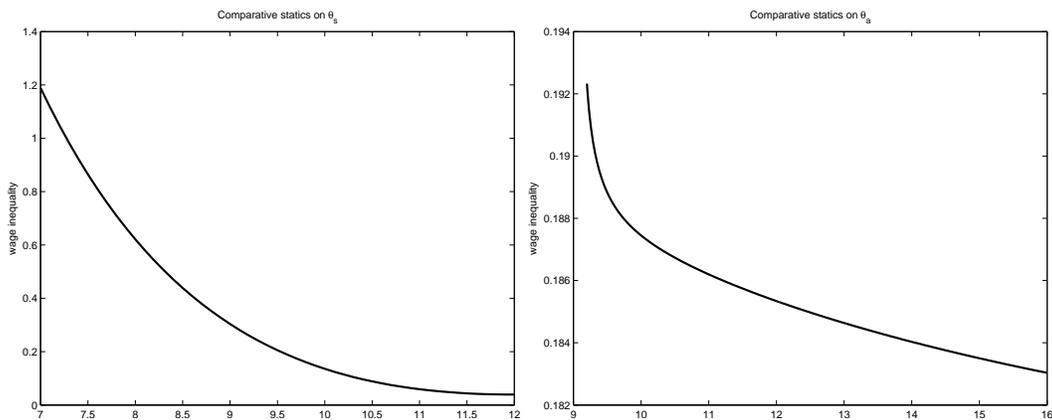


Figure 5.2: Comparative statics on  $\theta_s$  and  $\theta_a$

Figure 5.2 shows the result of comparative statics on  $\theta_s$  and  $\theta_a$ . In particular, there is a negative effect of  $\theta_s$

and  $\theta_a$  on wage inequality. This result is consistent with the fact that low value in  $\theta_s$  and  $\theta_a$  imply high level of variance of  $s$  and  $a$  respectively.

### 5.3 Counterfactual analysis

In this subsection, I will do several counterfactual analyses. The first is to set the job specific productivity  $A_{ij}$  in 2000 as the value in 1990. The second one is to replace the job match ratio across jobs. Finally, to quantify the contribution of performance-pay channel and skill-match channel, I calibrate job specific skill-match parameters  $\theta_{sij}$  and the monitoring cost  $M$  in both the 1990 and 2000, then do counterfactual analyses on these two parameters.

Table 5.2: Counterfactual analysis on  $A_{ij}$

Wage inequality			Counterfactual analysis
data(1990)	data(2000)	model(2000)	$A_{ij} = A_{ij,1990}$
0.201	0.284	0.203	0.208 (+2.5%)

**Job specific productivity** Table 5.2 shows that replacing job specific productivity has not changed the wage inequality very much. In particular, the baseline model generates 0.203 and the value in counterfactual case is 0.208, that is, it increases the wage inequality by 2.5%. This result implies that the productivity distribution across jobs actually decrease the wage inequality. Since productivity difference across jobs contributes to between-job wage inequality, it is consistent with our accounting exercise that the between-job component has small effect on residual wage inequality for the highly educated.

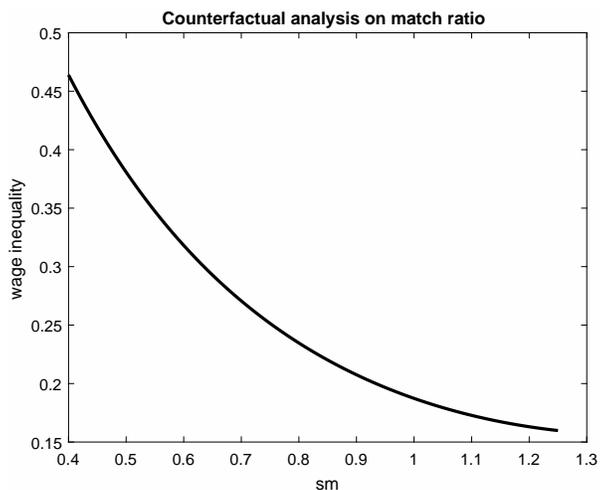


Figure 5.3: Counterfactual analysis on skill-match ratio

**Skill-match ratio** Figure 5.3 presents the counterfactual analysis on skill-match ratio. I change the skill-match ratio for all jobs by the same percentage and then compute the wage inequalities. For example, in the figure,  $sm = 0.8$  means the skill-match ratio is decreased by 20% for all jobs. It shows that the decrease of skill-match ratio will increase the wage inequality. This result is also consistent with our empirical facts that there is a negative correlation between job relatedness and within-job wage inequality.

**Job specific  $\theta_s$**  To better understand the skill-match channel, in this subsection, I will calibrate the job specific parameters  $\theta_{sij}$ . Since only the occupational information on skill match is available in the data, I decompose  $\theta_{sij}$  into the industry-component  $\theta_{si}$  and the occupation-component  $\theta_{sj}$ , that is,  $\theta_{sij} = \theta_{si}\theta_{sj}$ . I derive  $\theta_{sj}$  from data as the following way: from O\*NET and NLSY79 I computed the level of skill mismatch, and since high  $\theta_s$  implies low variance of skill mismatch, I use the inverse of skill mismatch as the value of  $\theta_{sj}$ . Then I calibrate industry specific  $\theta_{si}$  together with other parameters.

Table 5.3 shows a simple statistic description of  $\theta_{sij}$  in 1990. As shown in the table, the minimal and maximal value are 2.14 and 15.7 respectively and standard deviation is 3.13, hence it suggests a large variety of skill-match dispersion across jobs.

Table 5.3: Statistics of  $\theta_{sij}$

min	max	mean	sd
2.14	15.7	6.56	3.13

Based on the calibration result, I compute the wage inequality by assuming all jobs have the smallest value  $\theta_{smin}$  or the largest value  $\theta_{smax}$ . As shown in Table 5.4, if all the jobs have the minimal value, the wage inequality will increase to 3.43. This number is much higher than the value in the baseline model and it shows the potential of skill mismatch for explaining the increase of wage inequality.

Table 5.4: Counterfactual analysis on  $\theta_{sij}$

Wage inequality(1990)		Counterfactual analysis	
data	model	$\theta_{sij} = \theta_{smin}$	$\theta_{sij} = \theta_{smax}$
0.201	0.202	3.43	0.19

**Contribution to the change** To quantify the contribution of the performance-pay channel and skill-match channel to the change of wage inequality, I recalibrate  $\theta_{sij}$  and M in 2000 and then replace them with the values in 1990

respectively. Since  $M$  is the main parameter in the model determining the performance-pay incidence, in particular, low monitoring cost will induce high performance-pay incidence in a job, the contribution of  $M$  represents the contribution of performance-pay channel to wage inequality. In addition, the contribution of skill-match channel is represented by the contribution of  $\theta_{sij}$  though it determines the skill match partially.

Table 5.5 presents the main results in this paper. First of all, the model fits the data well: the wage inequality in data is 0.284 and in the model it is 0.286. The result shows that wage inequality in 2000 would decrease from 0.286 to 0.270 if  $\theta_{sij}$  are set to the levels in 1990. I compute the ratio of this change to the total change of wage inequality in the data, and conclude that skill match explains 18.8% of the increase of residual wage inequality for the highly educated. Since the  $\theta_s$  is job specific, it is hard to predict whether skill match is increasing or decreasing in the economy. However, in the comparative statics, I showed that the wage inequality is increasing in skill mismatch, generally speaking, skill mismatch is getting more serious among highly educated workers. Then the results can be interpreted as that 18.8% of the increase of wage inequality can be explained by the increase of skill mismatch in the economy.

Table 5.5: Counterfactual analysis on  $\theta_{sij}$  and  $M$

Wage inequality			Counterfactual analysis	
data(1990)	data(2000)	model(2000)	$\theta_{sij} = \theta_{sij,1990}$	$M = M_{1990}$
0.201	0.284	0.286	0.270 <b>(-18.8%)</b>	0.257 <b>(-34.1%)</b>

The analysis on performance-pay channel is similar. In particular, the wage inequality decreases to 0.257 when replacing  $M$  with the value in 1990. This result implies that the change of monitoring cost explains 34.1% of the increase of wage inequality. I compute the real monitoring cost by dividing job's productivity, that is,  $\frac{M}{A_1}$ . The calibration result shows that, in 2000,  $M=0.005$  and  $A_1 = 1.99$ , hence the real term is less than the value in 1990 which is 0.0035. Since low monitoring cost will generate high performance-pay incidence, the model predicts that generally performance-pay incidence has increased and it has contributed to the increase of wage inequality by 34.1%. This value is higher than the result in Lemieux et al. (2009) where the contribution from performance pay is 21%. This is mainly due to the fact that performance pay is more prevalent in the highly educated.

For sensitivity analysis, I calculate the contribution of  $\theta_s$  on different value of  $\theta_a$ . In this experiment, I deviate the value of  $\theta_a$  from baseline value and then compute the contribution of  $\theta_s$  on the change of wage inequality. The result on the left panel of Figure 5.4 shows that skill-match channel could have higher contribution when  $\theta_a$  is

getting smaller. In other words, when the ability distribution is getting more dispersed, skill match matters more in determining wage inequality. Similarly, the right panel of Figure 5.4 plots the contribution of  $M$  as the value of  $b$  changes. The result shows that as  $b$  increases  $M$  could have higher contribution to the change of wage inequality, that is, when the effort disutility is higher the contribution of performance pay will be higher.

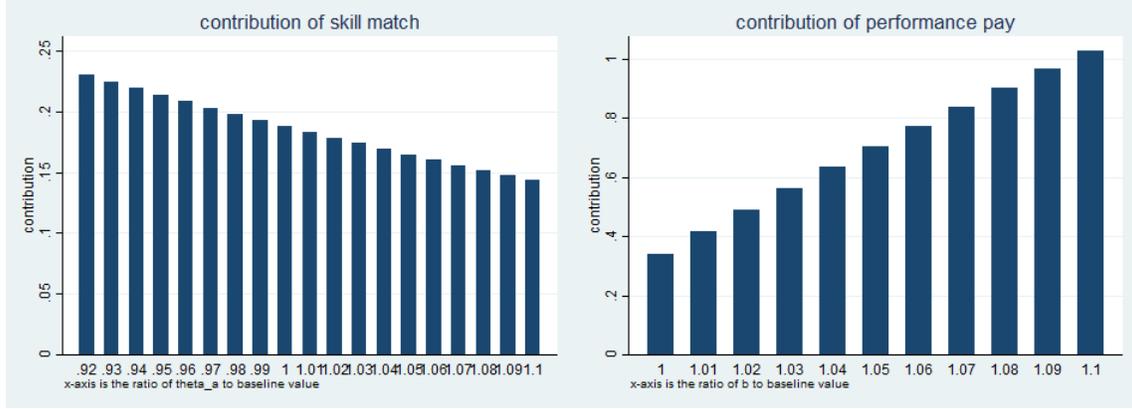


Figure 5.4: Contribution of  $\theta_{sij}$  as  $\theta_a$  change (Left), contribution of  $M$  as  $b$  change (Right)

## 6 Discussion: skill match on multiple dimension abilities

One way to extend the model is to include multiple dimension abilities. Specifically, workers' abilities are in multiple dimensions and jobs have different requirements or match premium on different dimensions. In this case, there is no way to rank people by abilities. People with high abilities in some dimensions may have low abilities in other dimensions. Hence as job becomes more specialized in tasks, wage inequality may change as well.

Following Sanders (2016), I model the match quality in the world of multiple dimension ability. Worker's abilities have  $N$  dimensions  $a = (a_1, \dots, a_N)$  and job  $g$  has skill requirement on  $N_g (\leq N)$  dimensions. Let  $N_g$  also the set of ability effective in job  $g$ , and the effective labor in job  $g$  is  $g = \{g_k | k \in N_g\}$ . Let time allocation  $(l_k)_{k \in N_g}$  then the effective labor in an unit time is  $h(a, g) = A_g [\sum_{k \in N_g} (a_k g_k l_k)^{\frac{\epsilon-1}{\epsilon}}]^{\frac{\epsilon}{\epsilon-1}}$ , where  $\epsilon$  is the elasticity of substitution between abilities in different dimensions. The optimal time allocation implies that  $h(a, g) = A_g [\sum_{k \in N_g} (a_k g_k)^{\epsilon-1}]^{\frac{1}{\epsilon-1}}$ . If there is only one dimension in both the ability and skill requirement, then  $h(a, g) = A_g a g$ , which is exactly the same as that in benchmark model. If there is only one type of ability but multiple job requirements then  $h(a, g) = A_g a [\sum_{k \in N_g} (g_k)^{\epsilon-1}]^{\frac{1}{\epsilon-1}}$ , and  $[\sum_{k \in N_g} (g_k)^{\epsilon-1}]^{\frac{1}{\epsilon-1}}$  is the match quality adjusted by optimal time allocation.<sup>9</sup>

<sup>9</sup>The case for multiple dimension for both side is complicated due to possibilities of combination.

**Job specialization** Job specialization is modeled as the decline of dimension of skill requirement. One special case is that jobs only specialize in the skill where they have comparative advantage. Therefore, if there is only one dimension in ability, then for same  $g$ , as  $N_g$  decreases the match quality will increase, and the between-job wage inequality will increase. But for within-job component there will be no change since match premium is the same to the workers. However, if there are multiple dimensions, then as  $N_g$  increase, a worker who matches with a job will have greater wage increase than those who are not matched, hence there is a change in within-job wage inequality as the economy becomes more specialized.

## 7 Conclusion

In this paper, I have documented that residual wage inequality for the highly educated has increased rapidly and that the main contributor to both the level and change is the within-job component. To explain this fact, I have built a model that allowed within-job wage inequality to be influenced by performance-pay incidence and skill mismatch. Workers with high ability choose to work in performance-pay positions and jobs are sorted by cutoff abilities. Skill mismatch increases wage inequality in that it affects joint output between workers and employers. The quantitative results are two-fold. The model explained around 71.5% of residual wage inequality for the high skill group in 2000. Skill-match channel explained 18.8% and performance-pay channel explained 34.1% of the increase in wage inequality from 1990 to 2000.

The model could be extended to include the underlying reasons for skill mismatch. In particular, a worker choosing to work in the unmatched job may do so because of preference change, promotion or search friction. Understanding the underlying reasons could have important policy implication. The model could also be extended to include multiple dimension abilities while studying the effect of job specialization on wage inequality. A third extension of this model would be to incorporate asymmetric information on a worker's ability or a job's characteristics.

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## A Appendix

### A.1 Derivation of expected utility on task $FP$ and $PP$ .

In task  $PP$ , workers solve the following problem

$$\begin{aligned} \max_e \quad & c - \frac{1}{2}be^2 \\ \text{s.t} \quad & c = (A\eta e - M)w. \end{aligned}$$

The the effort will be

$$e^P = \frac{A\eta a}{b}w,$$

and ex-post earning will be

$$E^P = \frac{(A\eta aw)^2}{b} - Mw,$$

which is increasing in  $a, \eta$  and decreasing in  $M$ . Ex-ante expected utility for worker  $a$  in this task is

$$EU^P(a) = \frac{(Aaw)^2}{2b}E(\eta^2) - Mw.$$

In Task  $FP$ , jobs will provide pooling wage to all the workers, based on the expectation of efficient labor in that task. Given effort level  $\bar{e}$ , the suggested earning is

$$E^F = E(A\eta a | a \in D_F)\bar{e}w$$

given the proposal, worker's optimal effort is

$$\bar{e} = \frac{E(A\eta a | a \in D_F)w}{b},$$

then earning will be

$$E^F = \frac{[E(A\eta a | a \in D_F)w]^2}{b},$$

ex-ante expected utility in task  $F$  is

$$EU^F = \frac{[E(A\eta a | a \in D_F)w]^2}{2b}.$$

## A.2 Sketch on solving model

### Firm's problem

#### 1. Optimal labor allocation

$$\left(\frac{H_{nF}}{H_{np}}\right)^{\gamma-1} = \frac{1}{c_n}, \quad n = 1, \dots, N$$

where

$$H_{np} = N_{np} \frac{A_n^2 w_n}{b} [p_n (Es^2 - 1) + 1] E_n [a^2 | a \in [a_n^*, a_{n+1}^*]], \quad n = 1, \dots, N$$

$$H_{nF} = N_{nF} \frac{A_n^2 w_n}{b} [p_n (Es - 1) + 1]^2 E_n^2 [a | a \in [1, a_1^*]], \quad n = 1, \dots, N$$

and

$$N_{np} = G(a_{n+1}^*) - G(a_n^*), \quad n = 1, \dots, N$$

#### 2. Labor demand

$$w_{ij} = \mu \beta_i \frac{Y}{Y_i} \left(\frac{Y_i}{Y_{ij}}\right)^{\frac{1}{\sigma}} Y_{ij}^{1-\frac{\gamma}{\mu}} H_{ijF}^{\gamma-1}, \quad (i, j) \sim n = 1, \dots, N$$

### Worker's problem

#### 1. Job sorting

In the equilibrium every job will have cutoff ability level which can be used to rank jobs, I use notation  $(i_n, j_n) \sim n$  to represent that job  $(i_n, j_n)$  has  $n$ th lowest cutoff value

#### 2. Self-selection

$$EU(a) = \begin{cases} E_n U^P(a) & a \in [a_n^*, a_{n+1}^*) \\ E_n U^F & a \in [1, a_1^*) \end{cases}, \quad n = 1, \dots, N$$

### Equilibrium conditions

#### 1. Equalization of expected utility in task F

$$E_n U^F = E_1 U^F = EU^F, \quad n = 1, \dots, N$$

Because of same expected utility, conditional on working in task  $F$ , the ability distribution is the same, hence

$$E_1[a|a \in [1, a_1^*]] = E_n[a|a \in [1, a_1^*]], n = 1, \dots, N.$$

## 2. Monotonicity of utility

$$E_{n+1}U^P(a) \geq E_nU^P(a), \text{ for } a \geq a_{n+1}^*, n = 1, \dots, N-1.$$

$$E_1U^P(a) \geq EU^F, \text{ for } a \geq a_1^*$$

## 3. Continuity of expected utility

$$E_1U^P(a_1^*) = EU^F,$$

$$E_{n+1}U^P(a_{n+1}^*) = E_nU^P(a_{n+1}^*), n = 1, \dots, N-1.$$

## 4. Labor market clear

$$\sum_n N_n = 1$$

$$N_n = N_{nF} + N_{np}, n = 1, \dots, N$$

**Equilibrium Equations** Since  $G(s) = 1 - (\frac{1}{s})^{\theta_s}$ ,  $G(a) = 1 - (\frac{1}{a})^{\theta_a}$ , denote  $a_{N+1}^* = +\infty$ ,

$$E_n[a|a \in [1, a_1^*]] = \frac{\theta_a}{\theta_a - 1} \frac{1 - (a_1^*)^{1-\theta_a}}{1 - (a_1^*)^{-\theta_a}}, n = 1, \dots, N$$

$$E_n[a^2|a \in [a_n^*, a_{n+1}^*]] = \frac{\theta_a}{\theta_a - 2} \frac{(a_n^*)^{2-\theta_a} - (a_{n+1}^*)^{2-\theta_a}}{(a_n^*)^{-\theta_a} - (a_{n+1}^*)^{-\theta_a}}, n = 1, \dots, N$$

$$Es = \frac{\theta_s}{\theta_s - 1}, Es^2 = \frac{\theta_s}{\theta_s - 2}.$$

Given the sorting order of cutoff abilities, I can pin down  $\{N_{np}, N_{nF}, H_{np}, w_n, p_n, a_n^*\}_{n=1}^N$  by following  $6N$  equations

$$N_{np} = (a_n^*)^{-\theta_a} - (a_{n+1}^*)^{-\theta_a}, n = 1, \dots, N \tag{A.1}$$

$$\left\{ \frac{N_{nF}[p_n(Es-1)+1]^2 E_n^2[a|a \in [1, a_1^*]]}{N_{np}[p_n(Es^2-1)+1] E_n[a^2|a \in [a_n^*, a_{n+1}^*]]} \right\}^{\gamma-1} = \frac{1}{c_n},, n = 1, \dots, N \quad (\text{A.2})$$

$$\sum_{n=1}^N N_{nF} = 1 - (a_1^*)^{-\theta_a} \quad (\text{A.3})$$

$$A_1 w_1 [p_1(Es-1)+1] = A_n w_n [p_n(Es-1)+1], n = 2, \dots, N \quad (\text{A.4})$$

$$\begin{aligned} \frac{1}{2} \frac{A_1^2 w_1}{b} [p_1(Es-1)+1]^2 E_1^2[a|a \in [1, a_1^*]] &= \left\{ \frac{1}{2} \frac{(A_1 a_1^*)^2}{b} w_1 [p_1(Es^2-1)+1] - M \right\} w_1 \\ \left\{ \frac{1}{2} \frac{(A_{n+1} a_{n+1}^*)^2}{b} w_{n+1} [p_{n+1}(Es^2-1)+1] - M \right\} w_{n+1} &= \left\{ \frac{1}{2} \frac{(A_n a_n^*)^2}{b} w_n [p_n(Es^2-1)+1] - M \right\} w_n, n = 1, \dots, N-1 \end{aligned} \quad (\text{A.5})$$

$$H_{np} = N_{np} \frac{A_n^2 w_n}{b} [p_n(Es^2-1)+1] E_n[a^2|a \in [a_n^*, a_{n+1}^*]], n = 1, \dots, N \quad (\text{A.6})$$

$$\begin{aligned} w_{ij} &= \mu \left\{ \prod_{i=1}^I \left\{ \sum_{j=1}^J [(c_{ij}^{\frac{\gamma}{1-\gamma}} + 1)^{\frac{\mu}{\gamma}} H_{ijp}^{\mu}]^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma}{\sigma-1} \beta_i} \right. \\ &\quad \cdot \beta_i \left\{ \sum_{j=1}^J [(c_{ij}^{\frac{\gamma}{1-\gamma}} + 1)^{\frac{\mu}{\gamma}} H_{ijp}^{\mu}]^{\frac{\sigma-1}{\sigma}} \right\}^{-1} \\ &\quad \left. \cdot [(c_{ij}^{\frac{\gamma}{1-\gamma}} + 1)^{\frac{\mu}{\gamma}} H_{ijp}^{\mu}]^{\frac{\sigma-1}{\sigma} - \frac{\gamma}{\mu}} \frac{H_{ijp}^{\gamma-1}}{c_{ij}}, (i, j) \sim n = 1, \dots, N \right. \end{aligned} \quad (\text{A.7})$$

### A.3 Sketch on calibration

1. The following equations will solve  $\{a_n^*\}$

$$\begin{aligned} a_1^* &= \left( 1 - \sum_{n=1}^N N_{nF} \right)^{-\frac{1}{\theta_a}} \\ a_{n+1}^* &= [(a_n^*)^{-\theta_a} - N_{np}]^{-\frac{1}{\theta_a}} \end{aligned} \quad (\text{A.8})$$

2. Given  $\{a_n^*\}$  the following equations will solve  $\{w_n\}, \{c_n\}, \{A_n\}$

$$w_1 = \frac{\frac{1}{2} \frac{A_1^2}{b} [p_1(Es-1)+1]^2 E_1^2[a|a \in [1, a_1^*]] + M}{\frac{1}{2} \frac{(A_1 a_1^*)^2}{b} [p_1(Es^2-1)+1]} \quad (\text{A.9})$$

$$w_{n+1} - w_n = \frac{A_1^2 w_1^2 (a_{n+1}^*)^2 [p_1(Es-1)+1]^2}{2bM} \cdot \left\{ \frac{(Es^2-1)p_{n+1}+1}{[p_{n+1}(Es-1)+1]^2} - \frac{(Es^2-1)p_n+1}{[p_n(Es-1)+1]^2} \right\}, \quad n = 1, \dots, N-1$$

$$A_n = \frac{p_1(Es-1)+1}{p_n(Es-1)+1} \frac{w_1}{w_n} A_1, \quad n = 2, \dots, N \quad (\text{A.10})$$

$$c_n = \left\{ \frac{N_{np}}{N_{nF}} \frac{[p_n(Es^2-1)+1]}{[p_n(Es-1)+1]^2} \frac{E_n[a^2|a \in [a_n^*, a_{n+1}^*]]}{E_n^2[a|a \in [1, a_1^*]]} \right\}^{\gamma-1}, \quad n = 1, \dots, N \quad (\text{A.11})$$

3. The following equations will pin down the earning share in occupations

$$E_j = \sum_{i=1}^I [(H_{ijp} - N_{ijp}M)w_{ij} + H_{ijF}w_{ij}], \quad (i, j) \sim n = 1, \dots, N \quad (\text{A.12})$$

where

$$H_{np} = N_{np} \frac{A_n^2 w_n}{b} [p_n(Es^2-1)+1] E_n[a^2|a \in [a_n^*, a_{n+1}^*]], \quad n = 1, \dots, N$$

$$H_{nF} = N_{nF} \frac{A_n^2 w_n}{b} [p_n(Es-1)+1]^2 E_n^2[a|a \in [1, a_1^*]], \quad n = 1, \dots, N$$

## B Tables and figures

Table B.1: Statistic description: NSCG1993

groups		observations	tenure	earning	inequality	proportion
gender	Female	34467	18.17	53814.12	0.20	0.37
	male	59893	19.73	76401.97	0.25	0.63
education	Bachelor	58063	18.77	61039.09	0.23	0.62
	Master	25757	20.17	68416.25	0.20	0.27
	PhD	10540	18.68	104475.17	0.28	0.11
race	White	79175	19.16	68444.02	0.23	0.84
	Black	9478	19.20	55785.14	0.19	0.10
	Hispanic	5707	17.27	62247.22	0.20	0.06
relatedness	Close	55613	18.96	70803.82	0.21	0.59
	Some	24066	19.04	67695.83	0.24	0.26
	Not	14681	19.69	57021.46	0.28	0.16
all sample		94360	19.11	67514.19	0.23	1

Table B.2: Statistic description: NSCG2003

groups		observations	tenure	earning	inequality	proportion
gender	Female	21180	20.13	60732.11	0.28	0.38
	male	34285	21.51	90929.36	0.38	0.62
education	Bachelor	31915	20.95	69964.33	0.34	0.58
	Master	16202	21.31	77226.22	0.28	0.29
	PhD	7348	19.72	131856.72	0.45	0.13
race	White	47212	21.07	79947.82	0.34	0.85
	Black	4411	20.70	61798.49	0.25	0.08
	Hispanic	3842	18.50	65085.11	0.33	0.07
relatedness	Close	33377	20.45	83487.92	0.30	0.60
	Some	13871	21.14	77929.15	0.34	0.25
	Not	8217	22.05	61807.33	0.41	0.15
all sample		55465	20.92	78042.82	0.34	1

Table B.3: Proportion of match degree: NSCG

groups		1993			2003		
		close	some	not	close	some	not
gender	Female	0.63	0.21	0.16	0.63	0.22	0.15
	Male	0.57	0.28	0.15	0.58	0.27	0.15
education	Bachelor	0.50	0.30	0.20	0.50	0.30	0.20
	Master	0.68	0.23	0.09	0.68	0.23	0.09
	PhD	0.88	0.08	0.04	0.87	0.10	0.04
race	White	0.59	0.26	0.15	0.60	0.25	0.15
	Black	0.60	0.22	0.18	0.59	0.24	0.17
	Hispanic	0.61	0.23	0.15	0.66	0.21	0.13
all sample		0.59	0.26	0.16	0.60	0.25	0.15

Table B.4: wage effect of job relatedness: NSCG(1993,2003)

VARIABLES	lnearnings 1990	lnearnings 2000
closely related	<b>0.171***</b> (0.00450)	<b>0.229***</b> (0.00687)
some related	<b>0.118***</b> (0.00450)	<b>0.170***</b> (0.00690)
exp	0.0361*** (0.000651)	0.0386*** (0.00101)
male	0.158*** (0.00332)	0.206*** (0.00498)
hgc	0.0681*** (0.00134)	0.0666*** (0.00212)
black	-0.0381*** (0.00620)	-0.0519*** (0.00924)
Constant	9.395*** (0.0256)	9.711*** (0.103)
Observations	92,802	55,039
R-squared	0.354	0.334

Standard errors in parentheses  
 \*\*\* p<0.01, \*\* p<0.05, \* p<0.1

Table B.5: Incidence of performance-pay jobs: PSID 1976-1998 (Source:Lemieux et al.(2009))

	Professionals	Managers	Sales	Clerical	Craftsmen	Operatives	Laborers	Services	Total
Min.&durables	0.46	0.60	0.74	0.35	0.21	0.26	0.26	0.19	0.33
Nondurables	0.52	0.62	0.79	0.38	0.26	0.42	0.25	0.06	0.43
Transport&utils.	0.23	0.52	0.82	0.28	0.27	0.37	0.44	0.37	0.35
Fin.insur.,&real est.	0.75	0.72	0.87	0.38	0.29	0.04	0.22	0.33	0.65
Bus.&prof.serv	0.41	0.57	0.73	0.39	0.46	0.40	0.18	0.25	0.43
Personal serv.	0.42	0.63	0.61	0.24	0.33	0.20	0.29	0.49	0.46
Whol-tr.&oth serv.	0.65	0.66	0.82	0.45	0.29	0.46	0.29	0.03	0.58
Retail trade	0.27	0.57	0.72	0.33	0.48	0.32	0.21	0.46	0.50
Construction	0.72	0.47	0.81	0.20	0.33	0.30	0.30	0.17	0.36
Agriculture&fishing	0.72	0.78	0.88	0.24	0.16	0.42	0.45	0.77	0.46
Total	0.45	0.59	0.78	0.34	0.30	0.33	0.31	0.34	0.42

Table B.6: Wage Losses from Mismatch

Mismatch Degree	Mismatch effect		
(High to Low)	5 years	10 years	15 years
90%	-0.052	-0.088	-0.124
70%	-0.034	-0.057	-0.080
50%	-0.023	-0.039	-0.054
30%	-0.015	-0.026	-0.036
10%	-0.008	-0.014	-0.020

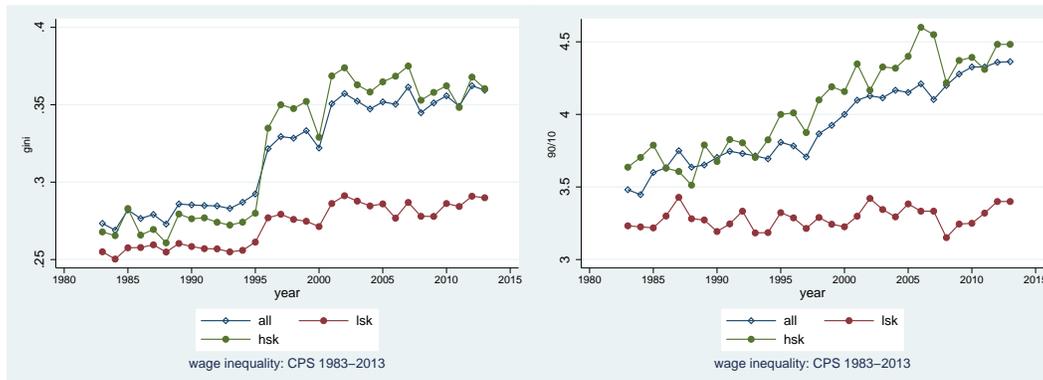


Figure B.1: wage inequality: Gini & 90/10

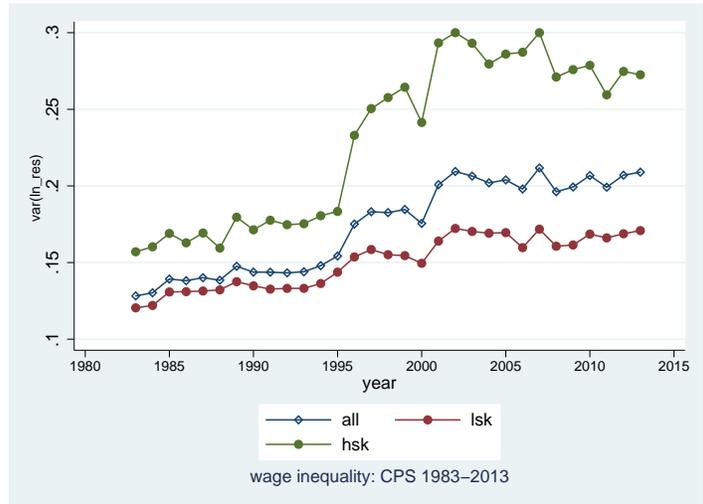


Figure B.2: wage inequality: log variance of residue